Ruth, as previously mentioned. However all the data do show an approximate logarithmic relationship between u and ϵ.

To determine whether there was an appreciable wall effect in any of the data it was decided to fluidize the largest spheres employed in the investigation, the 0.336 and 0.506-cm.-diameter spheres, in a column of diameter 2.603 cm. Comparison of these data with the data for the same spheres fluidized in the 5.508-cm.-diameter column should then give a measure of the maximum wall effect. Such a comparison is made in Figure 8, where porosity is plotted against Reynolds number, and it is concluded that wall effect is negligible.

SUMMARY

A careful experimental study, over a wide Reynolds number range from 0.005 to 1,800, was made of fluidization and sedimentation of glass spheres, with both water and ethylene glycol used. Porosities varied from about 0.50 to 0.91 and larger. Closely sized and spherical particles were obtained by grinding the particles between glass plates. For Reynolds numbers up to about 0.5 the data are in very good agreement with the laminar theory of Ruth, while the porosity function from Ruth's theory gave a satisfactory correlation for all the data, both laminar and turbulent. There were small but consistent differences between the fluidization and sedimentation data which indicated that for an expanded bed of given porosity more flow resistance is experienced in sedimentation than in fluidization.

ACKNOWLEDGMENT

This work was made possible by use of the facilities of the Ames Laboratory of the Atomic Energy Commission at Iowa State College. Invaluable assistance was also provided by Alden F. Presler.

NOTATION

D = particle diameter
D_c = column diameter
f = undetermined function of porosity
g = acceleration of gravity
K_u = function of particle and fluid properties, Reynolds number evalualted for Stokes's velocity [Equation (18)]
k = constant
p = permeability coefficient
u = fluidization velocity based on empty column cross section, or absolute velocity of top interface of bed in sedimentation
V = Stokes's velocity
ϵ = volume fraction of voids
μ = fluid viscosity
ρ = fluid density
δ = particle density

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Effect of Liquid on Interparticle Forces in Gas-fluidized Beds

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In an investigation of the behavior of an air-fluidized bed of glass spheres under varying interparticle forces, the results obtained are explained by hypothesizing the coexistence of particulate and aggregative fluidization. As interparticle forces are increased, a greater portion of the particles are in aggregative fluidization, resulting in a decrease in bed height. In this study water added to the fluidizing air increased the interparticle forces. Up to 0.5 mass % water was used, with a fluidized bed of glass spheres 0.013 to 0.035 in. in diameter. The resulting decrease in bed height has been correlated by means of a theoretical equation for the increase in interparticle forces due to the added water.

The widely differing natures of liquid- and gas-fluidized beds were observed in early laboratory fluidization experiments and received the names *particulate* and *aggregative* fluidization respectively (*11*). It has been suggested that the cause of aggregative fluidization is the attraction of individual particles for each other, usually termed *inter-particle forces*. The need for further information concerning interparticle forces has been indicated by several authors. Gamson has stated that a shape factor is needed to correlate particle heat and mass transfer data taken in aggregative fluidization with similar data from particulately fluidized beds and fixed beds (*3*). Workers who have studied heat transfer from surfaces in contact with the fluidized bed have also suggested that the attraction of particles for each other may be a factor in the correlation of heat transfer data (*1, 10*).

Interparticle forces may be divided into two groups, those due directly to the motion of fluid through the solids, and those forces not directly caused by the flowing fluid. The first class of forces are due to two causes: Bernoulli forces and the tendency of a flowing fluid to seek the path of least resistance. The latter phenomenon is very well described by

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Morse (7). The most common forces involved in the second group are electrostatic. In practically all laboratory studies attempts are made to minimize electrostatic forces. Some other possible forces in this group are the magnetic attraction of particles for each other and surface forces resulting in sticking together of the particles owing to liquid on their surfaces. The latter one was chosen for study in this paper.

LIQUIDS AND FLUIDIZED BEDS

The behavior of small amounts of liquid in a gas-fluidized bed has not been studied extensively. Meissner and Mickley have used a fluidized bed to remove mists from gas streams; however they state only that nonporous particles had a life of only a few minutes before liquid adhering to the particles stuck them together and fluidization stopped (6). The commercial realization of fluid coking has made practical application of liquid injection into a gas-fluidized bed, but there is little information as to the behavior of the fluidized bed. United States patent 2,567,959 has been issued for a fluidized-bed evaporator.

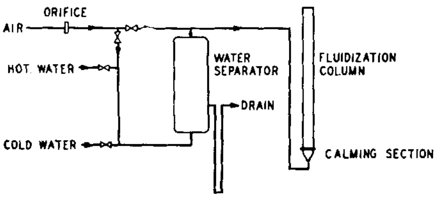

Fig. 1. Flow diagram of apparatus.

From the appearance of a gas-fluidized bed with liquid in it and a consideration of the forces involved, the action of interparticle forces caused by a liquid on the surface of the particles appears to be comparable to that resulting from forces due to other sources. It has been observed that the particles in dense-phase fluidization are in actual contact with each other (5, 3). Since the particles are in contact with each other, whether they are held together by a liquid film or other forces is not of great significance, provided that the forces are of the same order of magnitude.

For particulate fluidization to exist, the particles must be separated enough from each other so that interparticle forces are negligible; otherwise the particles would be attracted together, and aggregative fluidization would result. The type of interparticle forces present is not therefore important. It is realized that a liquid film on particles is not an effective interparticle force until the particles have actually touched, in contrast to other interparticle forces which act more or less continuously. However the very rapid decrease in attraction between particles as separation increases for other interparticle forces should allow the assumption of similarity for exploratory study.

Both particulate and aggregative fluidization may occur at the same time, resulting in dense- and light-phase fluidization. As interparticle forces are increased, a greater portion of the particles are in aggregative fluidization, resulting in a decrease in bed height. In the following section an equation is derived relating moisture content to bed height.

DERIVATION OF EQUATION RELATING BED HEIGHT TO MOISTURE CONTENT OF THE BED

For this derivation a hypothetical fluidized bed in which the interparticle forces may be varied independently of other conditions will be considered. When interparticle forces are negligible, particulate fluidization results. If the interparticle forces are sufficiently large, aggregative fluidization results. The difference in height between the fluidized bed in entirely particulate fluidization and entirely aggregative fluidization is proportional to the additional bed volume involved in particulate fluidization. The fraction of this excess bed volume occupied by the bed at a given set of conditions represents the fraction of particles in particulate fluidization.

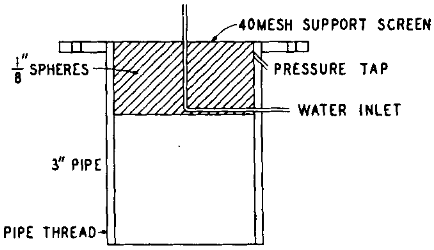

Fig. 1a. Calming section.

Fraction of particles in particulate fluidization

$$= \frac{h - h_m}{h_p - h_m} \qquad (1)$$

One minus the right-hand quantity in this equation is the fraction of particles in aggregrative fluidization. This assumes that the average bulk density of the two phases is independent of interparticle forces.

Equation (2) expresses in the least complex manner a relationship between the rate of particle aggregation and the fraction of particles in the particulate phase. The proportionality factor K_a is a function of the fluid, particle properties, and fluid velocity. The rate of particle aggregation is not however affected by the small quantities of water added during this study. At a maximum, 0.5 mass % water was added, and usually the addition was less than 0.1 mass %. This small amount of liquid on the particles cannot affect a fluidized bed until the particles have come in contact with each other.

Rate of aggregation

$$= K_a \frac{h - h_m}{h_p - h_m} \qquad (2)$$

A similar equation may be written for the rate of separation of particles from the aggregated phase, where the fraction aggregated is represented by one minus the fraction in the dispersed phase.

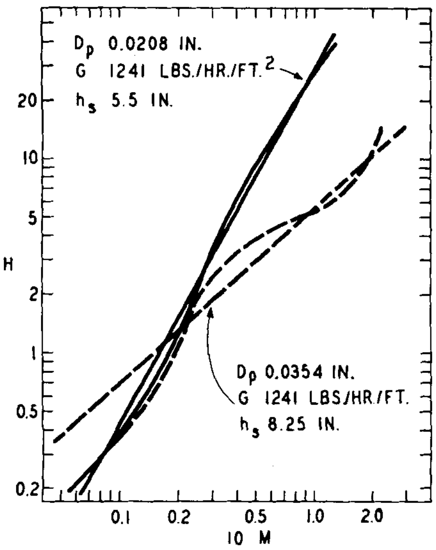

Fig. 2. H vs. M.

Rate of separation

$$= K_s\left(1 - \frac{h - h_m}{h_p - h_m}\right) \qquad (3)$$

In this case the proportionality factor is a function of moisture content. The forces holding the particles together, in this investigation, are the sum of two parts: those forces which are already present in the dry bed and the added forces due to the presence of water. Since the only quantity changed during a run was the amount of water present, the proportionality factor may be expressed in terms of constants as

$$K_s = \frac{K_1}{K_2 + f(M)} \qquad (4)$$

Equation (4) assumes that the dry interparticle forces and those due to the addition of water do not interact. The increase of interparticle forces due to added moisture content will be assumed to have the following relation:

$$f(M) = K_3 M^b \qquad (5)$$

Since the rates of agglomeration and separation are equal, Equations (2) and (3) are equal to each other and in com-

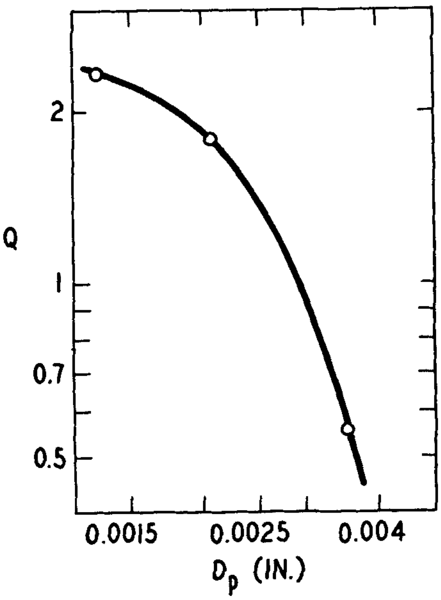

Fig. 3. Q vs. D_p.

bination with Equations (4) and (5) will give Equation (6):

$$K_a \frac{h - h_m}{h_p - h_m} = \frac{K_1}{K_2 + K_3 M^b} \cdot \left(1 - \frac{h - h_m}{h_p - h_m}\right) \quad (6)$$

This equation may be further simplified by algebraic manipulation and combination of constants to Equation (7):

$$\frac{h_p - h}{h - h_m} = K_4 + K_5 M^b \quad (7)$$

The term on the left side of Equation (7) may be expanded as follows to consider the height of a dry bed:

$$\frac{h_p - h}{h - h_m} = \frac{h_p - h_d}{h - h_m} + \frac{h_d - h}{h - h_m} \quad (8)$$

Equation (8) substituted into Equation (7) provides Equation (9).

$$\frac{h_p - h_d}{h - h_m} + \frac{h_d - h}{h - h_m} = K_4 + K_5 M^b \quad (9)$$

When the fluidized bed is dry, Equation (9) reduces to Equation (10).

$$\frac{h_p - h_d}{h_d - h_m} = K_4 \quad (10)$$

Equation (10) is substituted into Equation (9) to eliminate h_p.

$$\frac{K_4(h_d - h_m)}{h - h_m} + \frac{h_d - h}{h - h_m} = K_4 + K_5 M^b \quad (11)$$

Further manipulation and rearrangement following the steps shown below result in the final equation, (17).

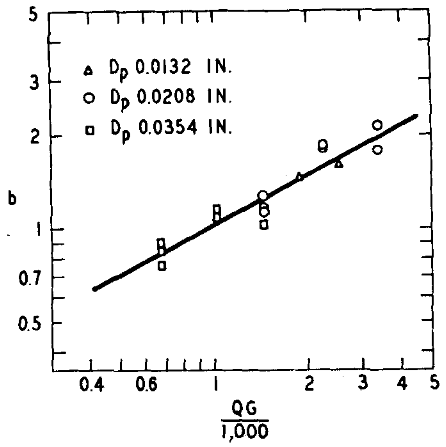

Fig. 4. b vs. $QG/1{,}000$.

$$\frac{h_d - h}{h - h_m} = K_4\left(1 - \frac{h_d - h_m}{h - h_m}\right) + K_5 M^b \quad (12)$$

$$\frac{h_d - h}{h - h_m} = K_4\left(\frac{h - h_d}{h - h_m}\right) + K_5 M^b \quad (13)$$

$$(K_4 + 1)\left(\frac{h_d - h}{h - h_m}\right) = K_5 M^b \quad (14)$$

$$\frac{h_d - h}{h - h_m} = K M^b \quad (15)$$

$$\frac{h_d - h}{h - h_m} \equiv H \quad (16)$$

$$H = K M^b \quad (17)$$

EXPERIMENTAL OPERATIONS

Figure 1 shows a sketch of the apparatus. The fluidization column was a 5-ft. length of 3-in. commercial Pyrex glass pipe. A detailed sketch of the calming section is also shown in Figure 1. It was necessary to both saturate the air used with water and to control its temperature, by injecting a mixture of hot and cold water into the air line and providing a tank for the air to separate from the water (Figure 1). The air entering the fluidization column was saturated with water and was slightly above room temperature. Cooling of this air by the surroundings offset warming of the air by passage through the fluidized bed and thus minimized drying of the bed.

The range of variables used is indicated in Table 1. A weighed quantity of particles (Superior Crystal Beads supplied by B. F. Drakenfield and Company) was charged to the apparatus and the desired air rate set. Distilled water could be added to the fluidized bed when nesessary by means of a metering pump, as shown in Figure 1. When the bed appeared in steady operation, its height was measured and a sample of particles taken. Samples taken simultaneously at various depths of the fluidized bed showed that no vertical moisture gradient existed. Samples of about 100 g. were required owing to the small moisture content. The quantity of water was determined by difference after the sample had been dried in an oven.

There was no large change in pressure drop across the fluidized bed as water was added, except with small particles and large amounts of liquid. In this case sufficient particles stuck to the wall of the apparatus to reduce the pressure drop appreciably. This difficulty was eliminated in taking data by tapping the side of the column, when necessary, to free the stuck particles. Moisture definitely increases the gas flow rate required for minimum fluidization and aids the formation of stable channels at lower velocities. Attempts to study these phenomena were not successful, because under some conditions the bed could alternate between stable channels and proper fluidization. To study this situation the moisture content of the bed would have to remain constant for a considerable period of time to determine its most usual mode of behavior.

ANALYSIS OF DATA

The following steps were taken in the evaluation of the constants in Equation (17). The experimental data were plotted on rectangular coordinates as mass percentage of moisture vs. bed height, and a curve which best fitted the points was drawn and transferred to logarithmic coordinates H vs. M, where M equals the analyzed mass percentage moisture minus 0.008. The subtractive term was an average residual moisture content in the fluidized bed. This residual moisture was present even when unsaturated air was used. When new beads were used, this term was smaller, but it soon reached this value after a few hours of operation. These used beads had a yellowish color which was insoluble in water or ether. Perhaps it was small adhering rust particles, which could have retained some moisture.

In an evaluation of H the measured height of the dry bed was used, but h_m needed to be found by trial and error, since it is approached asymptotically. The criterion used for the selection of h_m was the approach to a straight line of the plot of H vs. M on logarithmic coordinates. A straight line was drawn which best fitted the resulting curve, the slope and intercept of which gave values of b and K. Examples are given in Figure 2. This transfer of experimental data to logarithmic coordinates is shown as the curved lines in Figure 2. A straight line was then drawn which best fitted the curved line, the slope and intercept of which gave values of b and K. Table 2 presents the constants obtained in this manner. Equations for b and K as a function of particle diameter and air rate are

$$K = 185(100\, D_p)^{-2.68}$$

$$b = 1.02\left(Q\,\frac{G}{100}\right)^{0.54}$$

Q is presented graphically in Figure 3. These equations are compared with experimental values of b and K in Figures 4 and 5. Using these correlations and experimental values of h_d and h_m, one

TABLE 1. PROPERTIES OF MATERIALS USED

Material	Size no.	D_p, in.	Particle density, lb./cu.ft.	G_{mf}, lb./hr./sq. ft.
Glass spheres	3	0.0354	159	380
	7	0.0208	162	161
	10	0.0132	160	78.2

may plot Equation (17) on the original data. Figures 6 and 7 are examples of this. A reasonable correlation can be prepared for h_d and h_m, as shown in Figures 8 and 9, but owing to subtractive terms in the original equation it was desirable to use experimental values of h_m and h_d in the preparation of Figures 6 and 7. Figure 9 also includes portions of two runs of Lewis, Gilliland, and Bauer (*4*), which fall within the range of variables studied in this paper. The correlations of Figures 8 and 9 should not be used outside the range of data shown. More complex correlations such as those used by Lewis (*4*) are necessary for wider ranges of data.

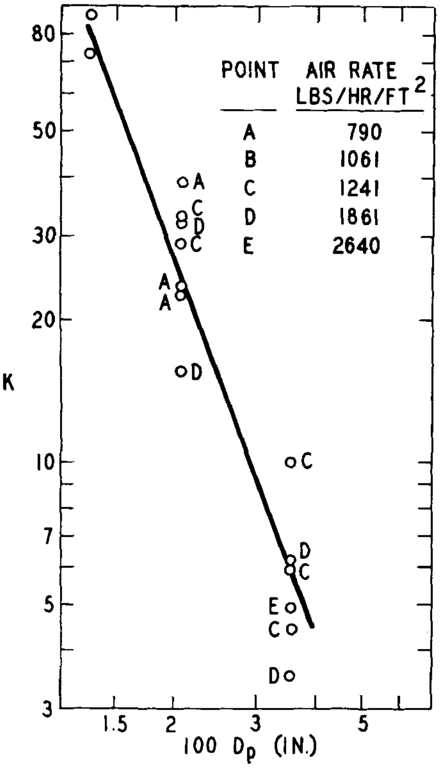

Fig. 5. K vs. D_p.

The ranges of variables employed in this study are presented in Table 1. The upper particle diameter was limited because larger glass spheres of the type used were not available. The lower limit was caused by the fact that the water on appreciably smaller particles makes fluidization impossible. The maximum bed height and air rates were controlled by the tendency of particles to be carried out of the system by entrainment. Fifteen different combinations of bed height, air rate, and particle size were run, over the possible ranges of moisture content, to give the original data on which the correlation is based (*8*). The equation constants are listed in Table 2.

The main reason for the scatter of the experimental data and the resultant scatter of points used in the correlation is the difficulty of measuring the fluidized bed height accurately. In this study it was done visually, since height had to be measured and a sample of the bed taken in a short period of time, owing to a slow drift in moisture content in the bed. At low moisture contents measurement of the water concentration also became a factor, resulting in scatter of the experimental data. Comparison of the experimental points and the curves drawn according to the theoretical Equation (17) is shown in Figures 6 and 7. For the four lines shown the average deviation of the data points is $\pm 7.7\%$. For the top line in Figure 6, obviously the worst of the cases presented, the average deviation is $\pm 14.5\%$. These data, typical of those obtained in the study, indicate reasonable agreement between experiment and theory, in view of the experimental difficulties encountered.

PROPOSED CRITERION FOR THE EXISTENCE OF AGGREGATIVE OR PARTICULATE FLUIDIZATION

Wilhelm and Kwauk have shown that Froude numbers $U_{mf}^2/(D_p g)$ greater than 1 result in aggregative fluidization and less than 1 in particulate fluidization, for the range of their variables. They mention however that this criterion would predict particulate fluidization for small particles in gas systems, which they demonstrate did not happen (*11*).

Since the velocity in the Froude number above is taken as being the velocity required for minimum fluidization, this criterion does not consider the type of fluidization to be a function of bed expansion. A fluidized bed of puffed rice which was aggregatively fluidized at low velocities became particulate at higher velocities, resulting in about a hundredfold bed expansion. For this case the Froude number was about 1.6. Aggregative fluidization becoming particulate in a liquid system was also noticed by Richardson and Zaki (*9*), who used ¼-in. steel spheres, cylinders, and cubes.

The Froude number was not intended to be used as a criterion for the type of fluidization when artificial interparticle forces are applied, and as discussed above is not applicable to all other cases of fluidization. The equations developed in this paper may give a basis for the prediction of a predominance of aggregative or particulate fluidization, in a particular case.

$$\frac{h - h_m}{h_p - h_m} = \frac{K_1}{K_a(K_2 + K_3 M^b)} \cdot \left(1 - \frac{h - h_m}{h_p - h_m}\right) \tag{6}$$

As shown in the derivation, the quantity on the left side represents the fraction of particles in particulate fluidization and that in parentheses on the right side represents the fraction of particles in aggregative fluidization. The quantities

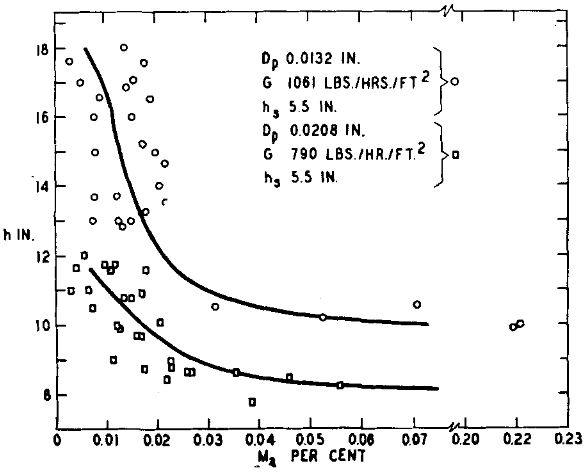

Fig. 6. h vs. M_a.

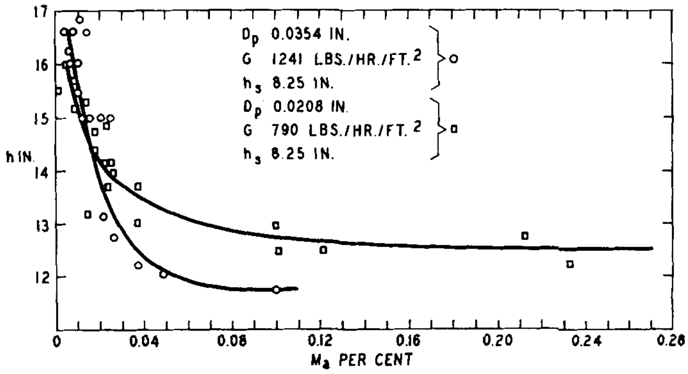

Fig. 7. h vs. M_a.

TABLE 2. SUMMARY OF EQUATION CONSTANTS

D_d, in.	G, lb./(hr.)(sq. ft.)	h_s, in.	h_m/h_s	h_d/h_s	b	K
0.0132	790	5.5	1.64	2.91	1.415	88.0
0.0132	1,061	5.5	1.82	3.27	1.6	75.0
0.0208	790	2.75	1.51	2.18	1.27	39.0
0.0208	790	5.5	1.44	2.09	1.14	23.5
0.0208	790	8.25	1.40	2.00	1.17	22.5
0.0208	1,241	2.75	1.87	3.18	1.83	33.0
0.0208	1,241	5.5	1.81	3.00	1.83	29.0
0.0208	1,861	2.75	2.18	4.73	2.14	32.0
0.0208	1,861	5.5	2.38	3.82	1.79	15.5
0.0354	1,241	2.75	1.49	2.18	0.76	10.0
0.0354	1,241	5.5	1.56	2.18	0.84	4.4
0.0354	1,241	8.25	1.49	1.94	0.91	5.9
0.0354	1,861	2.75	1.78	3.24	1.15	6.2
0.0354	1,861	5.5	2.00	3.04	1.07	3.5
0.0354	2,640	5.5	2.45	3.68	1.028	4.9

not in parentheses on the right side may be considered, as a group, to be analogous to a chemical equilibrium constant. When this equilibrium constant is very large, particulate fluidization results, and when it is small, aggregative fluidization is predominant. In this study only the portion of the equilibrium constant involving the moisture content was studied.

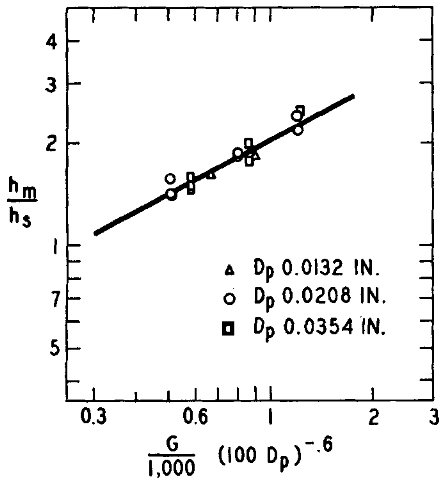

Fig. 8. $\frac{h_m}{h_s}$ vs. $\frac{G}{1{,}000}(100\ D_p)^{-0.6}$.

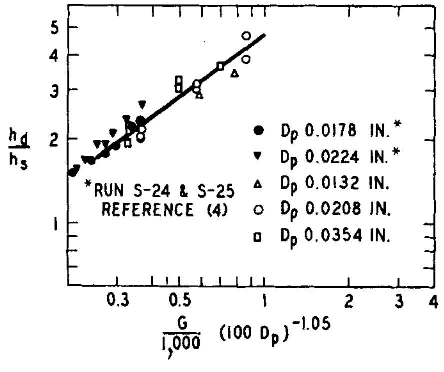

Fig. 9. $\frac{h_d}{h_s}$ vs. $\frac{G}{1{,}000}(100\ D_p)^{-1.05}$.

CONCLUSIONS AND RECOMMENDATIONS

The equation proposed in this paper relating bed height to liquid content has been shown to correspond to the experimental data. However the scatter of the data and the nature of the derivation made accurate quantitative results difficult to obtain.

The results which were obtained indicate that the height of a fluidized bed is little influenced by further additions of liquid after a initial small quantity has been added. Therefore if a maximum amount of liquid in a fluidized bed is desired, conditions should be selected to allow operation on the nearly horizontal portions of the height-vs.-moisture curve (Figures 6 and 7). Here other limiting factors concerning liquid in a fluidized bed, such as particles sticking to the wall or liquid being entrained out of the system by the gas stream, would occur.

A criterion has been suggested to indicate the predominance of either aggregative or particulate fluidization; however with the data available it was not possible to develop it completely.

Further study involving the independent variation of interparticle forces should aid in the extension of these concepts. By the addition of liquid to a gas-fluidized bed interparticle forces may be increased only in a system where they are already appreciable. A desirable extension of this work would involve increasing interparticle forces sufficiently from a negligible value, as in particulate fluidization, to cause predominately aggregative fluidization. One way of doing this would be by using particles which could be magnetized in a liquid system.

ACKNOWLEDGMENT

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NOTATION

- b = experimentally determined exponent in Equation (17)
- D_p = average diameter of particles fluidized, in.
- G = mass rate of air flow, lb./(hr.)(sq. ft.)
- G_{mf} = mass rate of air flow for minimum fluidization, lb./(hr.)(sq. ft.)
- H = by definition $(h_d - h)/(h - h_m)$
- h = height of fluidized bed, in.
- h_d = height of dry fluidized bed, in.
- h_m = height of bed when in complete aggregative fluidization, in.
- h_p = height of bed when entirely in particulate fluidization, in.
- h_s = height of settled bed, in.
- K = experimentally determined constant in Equation (17); when it is used with any subscript, a proportionality factor used in derivation of Equation (17)
- M = mass percentage of water added to fluidized bed
- M_a = mass percentage of water analyzed in fluidized bed
- Q = graphical function of D_p used to express b in terms of G and D_p in Figure 3
- U_{mf} = velocity for minimum fluidization, ft./sec.

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